THE 43rd—15th INTERNATIONAL—RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS
26 October—5 November 2012

1. The international SETI („Search for Extraterrestrial Intelligence”) collaboration is aimed at finding traces of life outside Earth. Recently a group of participating astronomers spotted an interesting flying object on the sky. Few details from the top secret report appeared in the daily press due to a witted journalist, triggering a considerable world wide panic:

   „... According to the measurements the unidentified flying object (UFO) approaches the Sun in the ecliptical plane. Analysis of the data reveals the fact that the trajectory of the UFO is such a parabola which allows the longest time for the object to spend within the (approximately circular) orbit of the Earth (assuming that it does not use at all its anticipated propulsion system). This interesting fact gives basis to the assumption that the flying object is a spacecraft sent by extraterrestial intelligence, with the purpose of gathering the maximum amount of information about terrestial life...”

   The key question we address in this problem is to find out all available information about the trajectory of the possible spy-UFO. We can assume during the calculations that all effects besides the gravitational force exerted by the Sun are negligible. Let us assume that the distance between Earth and Sun is constant, $R = 1$ astronomical unit.

   What is the minimal distance between the Sun and the UFO moving on the special optimal trajectory described in the UFO report? Moving at this optimal orbit how much time does the UFO spend within the orbital radius of Earth?

   (Péter Gnädig and Máté Vigh)

2. Two close friends, the thin Peter and the fat Paul arrive at a hostel, finding that only a lightly built bunk bed (with one upper and one lower place) is available. Peter wishes to have a good sleep, but he knows that his company moves around a lot while dreaming.

   Which bed should he choose to ensure a quiet sleep?

   He considers that taking the upper bed, the bed will move less by the movement of Paul, but will be amplified at the higher position. Taking the lower, the upper part where Paul sleeps of the bed will move more, but it is causing less displacement downstairs.

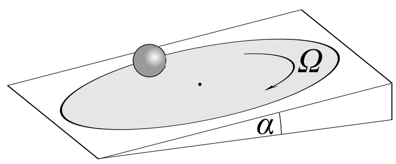
   Supported by a physicist's argumentation, let us give an advice to Peter!

   (Péter Gnädig)

3. If we lay down on our sides on flat ground, and start fluttering our upper leg back and forth parallel to the ground, it starts a periodic motion, sort of vibration, with a rather well defined frequency. If we lift the upper leg and start fluttering with the lower leg, the period of the motion will be considerably different. Try to explain the phenomenon with a simple model and determine the ratio of the frequencies!

   (Géza Tichy, co-founder of the Ortvay contest)

4. A planar, highly frictious disc is rotated with Ω angular velocity. The disc is inclined at α angle with respect to the horizontal plane. We are placing a small rubber ball on the disc, and we let it roll with some starting velocity in a certain direction and an initial angular velocity vector. Depending on these initial conditions, let us determine the possible trajectories of the center of the ball rolling on the disc! Can there be trajectories with special shapes (for example linear section or circle)? (The ball rolls without slip during the complete motion.)

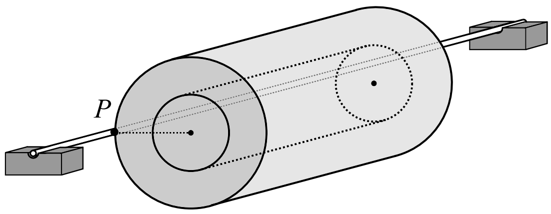
   (Máté Vigh)

5. (dedicated to the memory of Neil Armstrong)

   There is a concept to simplify the travel to Moon. A long, light rope is put on Earth orbit with two docking stations at both ends, and it is rotated such that when the closer station is a few hundred kilometers above the Earth surface its speed would match that of the surface, whereas the other end would travel at the escape velocity. At a given instance two spacecrafts of equal masses would attach to the rope: one launched from Earth and the other returning from Moon encounter. After half a turn, they would both detach. Determine the relevant parameters of such a space sling! Is it stable on orbit? What is the acceleration to be sustained by the astronauts during the travel?

   (András Bodor)

6. In a homogeneous, straight, solid cylinder a hole is prepared over its full length coaxially, such that the radius of the hole is half of that of the cylinder. An other cylinder, with same length and same material is placed precisely inside of the hole, such that it does not clog. The outer cylinder is fitted with a shaft along the line fitting to the point P which is on the outer side of the cylinder. The shaft has frictionless bearings and is placed horizontally. The system is released from an initial condition presented on the figure.



   a) What is the maximum value of the coefficient of static friction between the surfaces of the cylinders, if we find that the inner cylinder slips inside the outer one at the instant of the release?

   b) What should be the value of the coefficient of static friction such that the inner cylinder does not slip at all during the complete motion?

(László Pálfalvi)

7. A point mass m is hanged on an elastic thread, the gravitational acceleration is g. The elastic thread behaves like a spring of spring constant D_0 but exerts no force at all when unstretched (m, D_0 and g may be normalized to 1). The motion of the point mass is confined to the vertical line going through its equilibrium position; friction and drag is negligible. The mass is pulled $2mg/D_0$ under the equilibrium position and released.
   a) How much is the energy of the system (that is, the maximum of its kinetic energy during a period) and the period of the motion?
   b) The spring constant of the thread begins to vary *very slowly* (that is, $|\Delta D/D| \ll 1$ during a period). At a critical value of the spring constant the behaviour of the system changes qualitatively. Find this critical spring constant and the energy and the period of the system in the critical state.
   c) Describe the motion at the limit states $D \to 0$ and $D \to \infty$ qualitatively. Find the limit of the energy and the period in these states (if the limit is 0 or infinity, give the asymptotics as well). Examine some other quantities typical for the motion.

(Attila Szabó)

8. A moon of mass m is orbiting around a planet of mass M at a constant distance of R. The masses are of the same order of magnitude. From a direction perpendicular to the orbital plane of the system, a meteorite beam arrives, which is homogeneously distributed over an cross section much larger than R. All meteorite particles have the same initial velocity u. The double system forces the meteorites to deviate from the initial trajectories. Let us assume that the meteorites are fast, that is the orbital movement of the two large objects is negligible during the meteorite passage time. Let us calculate the scattering cross section, and study what kind of density pattern would emerge on an imaginary infinitely far „screen". What kind of „gravitational interference effect" is caused by the simultaneous presence of the two large objects?

(Gyula Dávid)

9. In the Lagrangian $L(\mathbf{r}, \mathbf{v}, t)$ of a point-like mass the velocity vector $\mathbf{v}$ appears only in the form of the unit vector $\mathbf{n} = \mathbf{v}/v$: $L(\mathbf{r}, \mathbf{n}, t)$, where $v = |\mathbf{v}|$ is the absolute value of the velocity vector.

   Derive the equations of the motion, and study the properties of the possible solutions! Let us try to convert the equations of the motion to the form of „mass $\cdot$ acceleration $=$ something"! Which quantity appears instead of the conventional concept of mass? Let us try to find conservation laws! Let us study the case of motion in a central force field, that is, when in the Lagrangian only the absolute value $r = |\mathbf{r}|$ of the position vector $\mathbf{r}$ appears!

(Gyula Dávid)

10. Let us study the classical model of „oscillator with memory", defined by the following Lagrangian:

$$L(q, \dot{q}, t) = \frac{1}{2m}\dot{q}^2(t) - \frac{1}{2}k\, q(t) \int_0^t d\tau \chi(t-\tau)\, q(\tau).$$

   Let us determine the equations of the motion by the variation of the action, and also solve these equations analytically if the form of the memory function $\chi(t)$ is the one below:

$$\chi(t) = \begin{cases} \Gamma e^{-\Gamma t}, & \text{if } t \geq 0, \\ 0, & \text{if } t < 0, \end{cases}$$

   and $\Gamma > 0$!

(Gábor Széchenyi)

11. A linear system is constructed from $2N$ pointlike balls, each of mass m, which are connected by ideal springs with spring constants of D_1 and D_2. The masses at the two ends of the chain are connected to walls with springs with spring constants of of D_2. Let us determine the vibrational frequencies of the chain! Compare these frequencies with the frequencies in the case of an infinitely long chain! Study separately the eigenfrequencies and vibrational modes in the cases of $D_1 < D_2$ and $D_1 > D_2$!

(József Cserti, László Oroszlány and Gábor Széchenyi)

12. A steel rod is fixed on one end to the wall, the part which extends outside of the wall is $l = 20$ cm long. The cross section of the rod is rectangular, the width is $a = 1$ cm, the height is $b =0.4$ cm. We hit the end of the rod – which kind of sound it produces?

(author: Béla Nagy, 1881–1954, publisher: Gyula Radnai)

13. The Stirling engine is an external combustion engine, where the gas is heated and cooled in a closed volume, which provides the work during expansion and compression.

a) Let us assume at the first place that the whole volume of the gas is able to reach the maximum and minimum temperatures provided by the external heat storages, and also that the pressure is homogeneous at any moment over the whole volume. In this sense, the system realizes the following cycle: constant volume heating, isothermal expansion, constant volume cooling, isothermal compression. Calculate the mechanical efficiency of the cycle! How much is it smaller than that of the Carnot-cycle?

b) The central concept of efficient Stirling engines is that during the constant volume heating (or cooling) the heat is not taken (or given) only from (to) the heat storage, but from the so called regenerator: the gas is pressed through a dense structure, constructed from adjacent layers or porous material, a kind of a filter. The gas exchanges heat with the regenerator, being heated or cooled. The transfer requires negligible amount of energy, since the total volume does not change. Let us also neglect the heat conduction due to the considerable temperature gradient in the regenerator.

Prove that during one cycle the amount of heat taken from the regenerator is the same as the heat returned, therefore the average temperature of the regenerator does not change – this means that heat transfers towards the heat storages take place only during the isothermal sections! Prove that in this cycle the mechanical efficiency is the same as the efficiency of the Carnot-cycle!

c) A real regenerator is not perfect. Let us assume that the regenerator is able to provide only a fixed α fraction (let us say 90%) of the necessary heat (or to absorb this fraction during cooling). How much does the efficiency of the Stirling engine degrade?

d) Let us assume that the regenerator is ideal ($\alpha = 1$), however the gas is not able to reach the maximum temperature provided by the external heat storage, but heats to a temperature lower by ΔT. What is the reduction of the mechanical efficiency, and by which factor does the work done in one cycle reduce?

(Dezső Varga)

14. Let us consider an argumentation along three true statements:

a) The „natural" variables of the entropy S are the internal energy U, the volume V and the particle number N. Let us assume that V and N are constant.

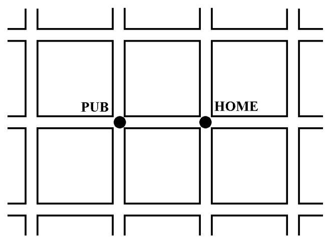
b) The entropy takes its maximal value in the equilibrium state.

c) The maximum of a function is where the first derivative is zero. Conclusion: $\frac{\partial S}{\partial U} = \frac{1}{T} = 0$, that is, in the equilibrium state the temperature of the system is infinite.

Where did this argumentation go wrong?

(XY)

15. A drunken fellow steps out of the door of the pub, realizing that he has completely lost his orientation abilities. He starts wondering in his very large city, which has rectangular street pattern (see Figure on the right). At each crossroads, after some consideration he chooses one of the four possibilities (without preference) and keeps walking along in this direction to the next junction where he chooses again randomly. If he is lucky enough to get home, which is indeed one block away from the pub, he wakes up in his own bed in the next morning. On the other hand if he returns to the pub after some wandering, he rather stays there for the rest of the night and wakes up on the pub floor at dawn.



a) Which is the probability that he returns home, or awakes at the pub – or that he manages to find neither, and the rising Sun finds him somewhere in the cornfields outside of the city?

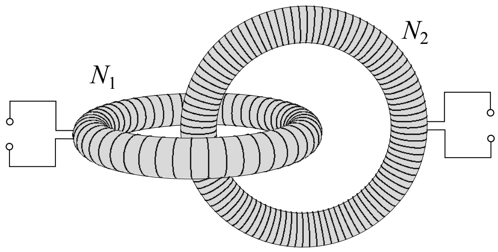
b) How do these probabilities change, if the short route connecting his home and the pub turns out to be blocked?

(Gábor Széchenyi)

16. A point charge q is located a distance r away from an infinitely long grounded conducting cylinder of radius a. Find the force acting between the charge and the wire, if $a \ll r$.

(Márton Lájer and Gábor Széchenyi)

17. There are two air core toroidal coils of the same sizes, entangled around each other according to the figure, differing only in the number of windings N_1 and N_2. The windings are uniformly gradual for both coils, their cross sections are A_1 and A_2, the radius of their mid-rings are $R_1 = R_2 = R$, and the mid-rings are perpendicular to each other.



a) What are the values of the self inductances (L_{11} and L_{22})?

b) What are the values of the coefficients of mutual inductances (L_{12} és L_{21})? Do they depend on the orientation of the winding?

c) Let us study whether the general conditions are fulfilled for the mutual inductance coefficient matrix elements L_{ik}! Discuss the case of $A_1 \ll A_2$.

(Péter Gnädig, Péter Vankó and Máté Vigh)

18. Is there an existing electric power line in Hungary, transferring electric current of $I = 2$ kA at $U = 400$ kV and at $f = 50$ Hz, where the radiation losses are comparable to the ohmic (resistive) losses?

(Gábor Etesi)

19. There are two apparently similar discs, one of them transmitting only the light with left circular polarization, the other only the light with right circular polarization. Let us determine which disc transmits which type of light, assuming that we can apply flat mirrors, semitransparent mirrors and a reasonably monochromatic light source (the semitransparent mirrors do not polarize the light). Let us assume that the positioning of the mirrors is possible to high precision, considerably better than the wavelength of light. If we get hold of two other discs which are linearly polarizing the light in a certain direction, are we able to determine the direction of this polarization using the above components?

(András Bodor)

20. Let us assume that neither the special, nor the general theory of relativity are valid, and we attempt to explain the observed phenomena according to the classical mechanics and electrodynamics! Let us assume on the other hand, that the gravitational field influences the propagation of light, such that the local speed of light c depends on the local gravitational potential Φ! Far from any objects the gravitational potential approaches zero, therefore we get back the usual value c_0 of the speed of light in vacuum. *a*) How should one choose the function $c(\Phi)$ such that our theory can reproduce the gravitational lensing effect known from the general theory of relativity? *b*) What should be $c(\Phi)$ if the lensing effect is reproduced only for the weak field approximation? Could we suggest experiments to decide whether our theory is correct?

(Gyula Dávid)

21. There is a spherically symmetric star in the center of the coordinate system with mass of M and radius of R, which gives rise to the well known Schwarzschild-type space-time metric all around. How much do the energy and momentum of the system (the star + the surrounding gravitational field) appear for a Minkowski-type inertial observer which is infinitely far from the star, and moves with constant velocity V with respect to the star? What is the total mass of the system?

(Gyula Dávid)

22. The specific spin state of a particle with spin $S = 1$ is unknown, but we can prepare and reproduce this state any times we wish. How many measurements are at least necessary to determine this common unknown state? Let us construct an appropriate series of measurements!

(Tamás Geszti)

23. An electron is placed in a magnetic field of constant magnitude and steadily varying direction along the z coordinate: $\mathbf{B} = (B\cos kz, B\sin kz, 0)$. Assume that the momentum of the electron is z-directed.
(a) What are the eigenstates of the electron and what is the energy of these states?
(b) What is the probability of measuring the electron spin parallel or antiparallel with vector $\mathbf{B}$ (varying with coordinate z) in an eigenstate?
(c) Let an electron of $+z$-polarized spin and momentum $(0, 0, p)$ be placed in such a magnetic field. When is it possible that the electron becomes $-z$-polarized after a while? How much time is needed for the polarization change in this case? If the magnetic field is switched off after this time, how much can be the momentum of the electron?

(Attila Szabó)

24. What should be the size ratio of Ga and As atoms in a GaAs semiconductor in order to get the tightest volumetric packing?

(Géza Tichy, co-founder of the Ortvay contest)

25. A particle with mass m moves in an one dimensional periodic potential (with period length of a). For the movement by one grid period a the trace of the transfer matrix $\mathbf{T}$ is given by the following equation (the explicit form of the transfer matrix is irrelevant for the rest of the problem):

$$\mathrm{Tr}\,\mathbf{T} = \frac{8}{\pi}\,\mathrm{Arcsin}\left[\cos\left(\frac{\pi}{4}\sqrt{\frac{E+\beta\,E_0}{E_0}}\right)\right]$$

In the above formula E is the energy of the particle, E_0 and β are positive real constant parameters, Arcsin means the principal value of the function.

Let us determine the band structure of the system! Give dispersion relation $E_n(K)$ of the n-th band as a function of the quasi-momentum K! Let us calculate the gap between the n-th and $(n+1)$-st band! Which is the key parameter determining whether the system will be insulator or conductor?

(Gyula Dávid)

26. We are attempting to construct a device which accelerates muons. Let us disregard the numerous details (how to produce the muons, how to form a particle beam, how to keep them on trajectory, how to realize the collisions etc), but concentrate on the issue of maximum achievable energy in the acceleration process.

Let us assume that the muons are initially at rest, and the acceleration takes place along the distance of L (assume that the particles run along L only once – at this point it is irrelevant if the particles are in fact run around a small ring multiple times). The accelerating electric field E_0 is constant along the trajectory, that is the terminal muon energy is eE_0L.

The main problem of efficient acceleration is the decay of the muons. Let us calculate which fraction of the muons would reach the end of the L long trajectory! Give this survival rate as a function of E_0 and the achieved terminal energy!

Discuss the results from the practical point of view. With contemporary technologies the value of E_0 is in the order of 100 MV/m. Which is the achievable maximum muon beam energy if we require that the surviving muon fraction is, let's say, one in a thousand? Would it be possible to study the production of the Higgs particle in the collision of such beams, and if not, which is the necessary value of E_0 to reach the sufficiently high energy (roughly twice the rest mass of the Higgs particle)?

How big this accelerator would be? Let us assume that we can construct bending magnets with maximum field of $B_0 = 5$ T. Which is te radius of the storage ring? Is it true that the energy loss by synchrotron radiation is much smaller than the power input during acceleration?

(Dezső Varga)

27. Consider a device where in a vacuum-filled domain, decaying nuclei emit electrons in random directions, with a given kinetic energy that is much smaller than the rest energy of the electron. In this domain there is an almost homogeneous, strong B_s magnetic field pointing in the z direction, which forces the emitted electrons onto a helical path around the field lines. Thus the electrons will follow the field lines. The strength of the magnetic field along the field lines decreases slowly (ie. the surface occupied by the field lines increases, as the field lines slowly diverge), and the electrons arrive in a domain where homogeneous B_w magnetic field is present, which points in the z direction, and is much weaker than B_s (possibly by many orders of magnitude).

We assume that the kinetic energy of the electrons is small, so the cyclotron radius corresponding to the fast orbital motion of the electrons is much smaller than the typical distance scale of the variation of the magnetic field. So we are looking for the leading order effect.

The decay process is isotropic, therefore initially (as it is easy to show) the distribution of the p_z momentum component of the electrons is uniform.

Let's determine the distribution of the momentum component p_z of the electrons in the domain of the weak magnetic field! What is the cause of the dramatic change? Determine the distribution of the angle between the momentum of the electrons and the magnetic field in this domain! In some appropriate manner, define the 'width' of this angular distribution and of the p_z distribution (eg. by half-width, or if exists, the variance), and express these quantities in terms of the B_w and B_s field strengths and of the other geometrical parameters!

(Márton Nagy and Dezső Varga)

28. A graph is called a star if there is a node called the central node, and nodes on the perimeter such that the central node is connected to every other node but there are no links between nodes on the perimeter. The number of nodes on the perimeter is N. We can define a simple Ising-model on the graph. The index of the central node is 0, then the energy of the spin configuration $s = (s_0, s_1, \ldots, s_N)$ is

$$H_N(s) = -J \sum_{i=1}^{N} s_i s_0 - h \sum_{i=1}^{N} s_i - h s_0.$$

If the inverse temperature of the environment is β, the probability of finding the system in a state with spin configuration s is

$$p_N(s) = \frac{e^{-\beta H_N(s)}}{\mathcal{Z_N}},$$

where $\mathcal{Z}_N$ is the usual partition function. The density of magnetisation on the perimeter is $m(s) = N^{-1} \sum_{i=1}^{N} s_i$. The problem is the following. Given a smooth function $f(z)$, calculate the

$$\lim_{h \to 0^+} \lim_{N \to \infty} \langle f(m) \rangle_N$$

thermodynamical average, where $\langle f(m) \rangle_N$ is the thermal average of the function f of the density of magnetization on the perimeter with the Boltzmann distribution given above. The order of the limits is crucial in getting the right answer.

(Norbert Barankai)

29. Felix Baumgartner, who weights 80 kg, bravely falls from the altitute of 39 km, opens the parachute at 1527 m and lands safely.

After the jump, how much time it takes and at which altitude happens that his speed reaches the local sound velocity? When and at which altitude do he slow down such that he resumes to velocity below that of the sound?

Let us plot the $\beta(t) = v(t)/c(t)$ ratio as a function of altitude between 39 000 m and 1527 m, where $v(t)$ and $c(t)$ are the momentary speed and the local sound velocity as a function of time. Is it possible that he crossed the sound velocity multiple times? Is it true that the maximum of $\beta(t)$ matches with that of his momentary velocity $v(t)$?

During the calcluations, let us take the $T(z)$ temperature and $c(z)$ sound velocity values as a function of altitude from the plot given below: `http://en.wikipedia.org/wiki/Speed_of_sound`

The altitude dependence of the pressure $p(z)$ may be determined by the barometric altitude formula, where take the gravitational acceleration $g = 9.8\ \mathrm{m/s^2}$, the pressure at sea level of $p_0 = 10^5$ Pa, and the air density at sea level to be $\rho_0 = 1.225\ \mathrm{kg/m^3}$. Let the air density as a function of altitude be

$$\rho(z) = \frac{p(z)\, M}{R\, T(z)},$$

where M is the molar mass of air, R is the universal gas constant. Let us assume that the atmospheric drag exerts a force $F = 1/2 \cdot A \cdot C \cdot \rho(z) \cdot v^2(z)$ also above the sound velocity, disregarding the sound barrier!
Here $C \approx 0.4$ is the shape factor in the drag force, and let us assume that the effective cross section is $A = 1\ \mathrm{m}^2$.

To what extent and by which amount this model differs from reality?

(Gábor Homa and Gábor Tóth)

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